# LUMINOUS RADIATION FROM A BLACK BODY AND THE MECHANICAL EQUIVALENT OF LIGHT

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CONTENTS



## I. INTRODUCTION

Numerous investigations have been made to determine the relation between visual sensation, "light," and radiant power, and, more specifically, the radiant power required to produce an intensity of 1 candle.

In order to calculate data on visual sensation from the radiation emitted by a black body, it is necessary to have a mathematical equation of the visibility curve of the average eye. Such an equation has been given in a preceding paper,[1] [a] which includes experimental data on the average eye representing 125 observers. This equation, which fits the observations very closely, is of the form

$$V_\lambda = V_m R^n e^{n(1-R)} \tag{1}$$

In this equation $V_\lambda$ represents the visibility of radiation at a given wave length relative to the visibility, $V_m$, at the wave length of maximum visibility, and $R=\frac{\lambda_m}{\lambda}$.

In order to make this equation fit the observed data, it has to be used in the following form:

$$\begin{aligned} V_\lambda = 0.999(R_1 e^{(1-R_1)})^{200} + 0.035(R_2 e^{(1-R_2)})^{400} \\ +0.130(R_3 e^{(1-R_3)})^{1000} + 0.084(R_4 e^{(1-R_4)})^{2000} \end{aligned} \tag{2}$$

[a] See bibliography at conclusion of paper for notes referred to throughout text by superior figures.

In this formula $R_1 = \frac{0.556}{\lambda}$, $R_2 = \frac{0.455}{\lambda}$, $R_3 = \frac{0.610}{\lambda}$, and $R_4 = \frac{0.525}{\lambda}$, where $\lambda$ is expressed in microns.

## II. LUMINOUS FLUX EMITTED BY A BLACK BODY

The visibility curve of the average eye and the mathematical equation representing it are of interest in computing the brightness (the luminous flux emitted), the luminous efficiency, etc., of a black body as a function of its temperature. Extensive computations of this type have been made by various writers, the most recent papers being by Nutting,[2,3] Pirani,[4] Foote,[5] Kingsbury,[6] etc. In many cases the numerical data were obtained by graphical multiplication and integration. For example, one could obtain the luminous efficiency by computing the spectral energy curve of a black body at a given temperature, using Planck's equation. This curve is drawn to scale, and from it a new curve is obtained in the visible spectrum by laying off ordinates proportional to the product of the ordinates of the original curve and the visibility curve. The ratio of the areas of these curves obtained by graphical integration gives the luminous efficiency. These graphical computations may be avoided by following the procedure of Kingsbury,[6] who has reduced these data by mathematical integration. In the present paper only a few of these computations will be given in order to show the modification of Kingsbury's numerical values when using the most probable radiation constants [7] and the new visibility curve. For this purpose it is of interest to recall that the luminosity $L_\lambda$ of a black body is the product of the energy, $E_\lambda$, emitted per second and the visibility at each wave length. The integral luminosity ("luminous power," "brightness," "Helligkeit," etc., are terms commonly used when considering the light emitted) per unit area is, therefore,

$$L = \frac{1}{\pi}\int_0^\infty V_\lambda E_\lambda \, d\lambda \tag{3}$$

in which $V_\lambda$ is obtained from equation (1) and $E_\lambda$ from Planck's equation $E_\lambda = c_1\lambda^{-\alpha}(e^{c_2/\lambda T} - 1)^{-1}$. The value of $L$ is in light watts per steradian per square centimeter (projected area) of the radiating surface. The factor $\frac{1}{\pi}$ is introduced in order to give the normal flux density, $\sigma_0$, of radiation per cm$^2$ per deg.[4] The total hemispherical flux, on this basis, is $\sigma = \pi\sigma_0$.

In order to calculate the luminosity mathematically,[6] it is necessary to transform equation (3) so that it can be integrated exactly. For this purpose the part of the Planck equation which is in parentheses is expanded by the binomial theorem giving:

$$E_\lambda = c_1\lambda^{-\alpha}(e^{c_2/\lambda T} - 1)^{-1} = c_1\lambda^{-\alpha}[e^{-c_2/\lambda T} + e^{-2c_2/\lambda T} + \cdots \cdot e^{-mc_2/\lambda T}] \quad (3a)$$

In view of the fact that we are interested in evaluating the energy in the visible spectrum, only the first term, $e^{-c_2/\lambda T}$, in the binomial series, equation (3a), which corresponds with the Wien equation, needs to be used in equation (3).

Writing equation (1) in the form $V_\lambda = \left(\frac{\lambda_m}{\lambda}\right)^n \frac{e^n}{e^{n\lambda_m/\lambda}}$ and placing $V_m = 1$, equation (3) becomes

$$L = \frac{c_1 e^n \lambda^n{}_m}{\pi} \int_0^\infty \left[\lambda^{-n-\alpha} e^{-\left(n\lambda_m + \frac{c_2}{T}\right)\frac{1}{\lambda}}\right] d\lambda \quad (3b)$$

By placing $\lambda = \frac{1}{\lambda'}$, equation (3b) becomes

$$L = \frac{c_1 e^n \lambda^n{}_m}{\pi} \int_0^\infty \left[\lambda'^{(n+\alpha-2)} e^{-\left(n\lambda_m + \frac{c_2}{T}\right)\lambda'}\right] d\lambda' \quad (3c$$

This integral has the form of the gamma function,

$$\int_0^\infty x^p e^{-kx} dx = \frac{\Gamma(p+1)}{K^{p+1}} = \frac{\Gamma(n+\alpha-1)}{K^{p+1}}$$

in which

$$p = n+\alpha-2 \text{ and } K^{p+1} = \left(n\lambda_m + \frac{c_2}{T}\right)^{n+\alpha-1}$$

$$= (n\lambda_m)^{n+\alpha-1}\left(1 + \frac{c_2}{n\lambda_m T}\right)^{n+\alpha-1}$$

Hence equation (3c) becomes:

$$L = \frac{c_1 e^n \lambda^n{}_m \Gamma(n+\alpha-1)}{\pi(n\lambda_m)^{n+\alpha-1}\left(1 + \frac{c_2}{n\lambda_m T}\right)^{n+\alpha-1}} \quad (3d)$$

Using Stirling's formula (see English edition of Planck's "Heat radiation," p. 218) for large values of $n$, we have

$$\Gamma(n+\alpha-1) = (n+\alpha-2)! = \frac{\sqrt{2\pi}}{e^{n+\alpha-2}}(n+\alpha-2)^{n+\alpha-3/2}$$

The value of $c_1$, which appears in equation (3d), is obtained from the relation (using $\alpha = 5$)

$$c_1 = \frac{\sigma T^4}{\int_0^\infty \lambda^{-5}(e^{c_2/\lambda T} - 1)^{-1} d\lambda} = \frac{\sigma c_2^4}{6.494}$$

This value is obtained by using the binomial expansion of the Planck equation given in equation (3a), which is integrated term by term, after which the whole is summated[b] from $m = 1$ to $m = \infty$ for $\alpha = 5$.

Substituting these values, equation (3d) becomes

$$L = \frac{\sigma}{\pi} \frac{A}{\left(1 + \frac{B}{T}\right)^{n+4}} \tag{4}$$

where

$$A = 0.01922 \left(\frac{c_2}{\lambda_m}\right)^4 \frac{1}{\sqrt{n}} \left(\frac{n+3}{n}\right)^{n+7/2} \text{ and } B = \frac{c_2}{n\lambda_m}$$

Using an equation containing four terms [equation (2) instead of equation (1)] for $V_\lambda$ and using the constants $\lambda_m = 0.556$, $0.445$, $0.610$, and $0.525\mu$, respectively, $c_2 = 14\,350$ and $\sigma = 5.7 \times 10^{-12}$ watt per cm² per deg.[4], the luminous intensity (in light-watts per steradian per cm²) of a black body at an absolute temperature $T$ is

$$L = \frac{5.7 \times 10^4}{\pi} \left[ \frac{1.247}{\left(1 + \frac{129.05}{T}\right)^{204}} + \frac{0.0678}{\left(1 + \frac{78.85}{T}\right)^{404}} + \frac{0.0489}{\left(1 + \frac{23.52}{T}\right)^{1004}} + \frac{0.0406}{\left(1 + \frac{13.67}{T}\right)^{2004}} \right] \tag{5}$$

These data are given in Table 1. If we have the visibility constant, $V_m$, which is the ratio of the candle (or lumen) to the watt at the wave length of maximum visibility, these values of $L$ can be transformed to candlepower. For this purpose in equation (3) $L = bm$, where $b$ = brightness in candlepower per unit area and $m = \frac{1}{V_m}$ = the least mechanical equivalent of light, which is discussed on a subsequent page.

[b] See Preston's Theory of Heat (revised by Cotter), p. 607, 1904, for the derivation of the constants $c_1$ and $c_2$.

TABLE 1

**Data on the Luminous Intensity of a Black Body and the Mechanical Equivalent (m) of Light**

| T, degrees absolute | Luminous intensity L watt/cm² | Total intensity $\sigma_0 T^4$ watt/cm² | Radiant luminous efficiency | Brightness b, in candles per cm² (Hyde) | m=L/b | Crova wave length |
|---|---|---|---|---|---|---|
| | | | | | | $\mu$ |
| 1200 | $2.34\times10^{-5}$ | 3.762 | 0.00000622 | ........... | ........... | 0.605 |
| 1600 | $3.45\times10^{-3}$ | 1.189 | .000290 | ........... | ........... | .591 |
| 1700 | $8.46\times10^{-3}$ | $1.515\times10$ | .000558 | 5.2 | 0.001627 | .589 |
| 1800 | $1.88\times10^{-2}$ | $1.905\times10$ | .000987 | 11.3 | .001664 | .586 |
| 1900 | $3.85\times10^{-2}$ | $2.365\times10$ | .00163 | 23.3 | .001652 | .584 |
| 2000 | $7.34\times10^{-2}$ | $2.903\times10$ | .00253 | 45.0 | .001631 | .582 |
| 2100 | $1.32\times10^{-1}$ | $3.529\times10$ | .00374 | 81.0 | .001630 | $.580_5$ |
| 2200 | $2.26\times10^{-1}$ | $4.250\times10$ | .00532 | 138.0 | .001638 | .579 |
| 2300 | $3.69\times10^{-1}$ | $5.077\times10$ | .00727 | 227.0 | .001626 | $.577_5$ |
| 2400 | $5.79\times10^{-1}$ | $6.020\times10$ | .00962 | 359.0 | .001613 | .576 |
| 2500 | $8.77\times10^{-1}$ | $7.087\times10$ | .0124 | 550.0 | .001595 | .575 |
| 2600 | 1.29 | $8.291\times10$ | .0156 | 810.0 | .001593 | .574 |
| 3000 | 4.66 | $1.470\times10^{2}$ | .0317 | ........... | (c) | .570 |
| 4000 | $3.85\times10$ | $4.645\times10^{2}$ | .0829 | ........... | ........... | .563 |
| 5000 | $1.36\times10^{2}$ | $1.134\times10^{3}$ | .1201 | ........... | ........... | .559 |
| 6000 | $3.26\times10^{2}$ | $2.351\times10^{3}$ | .1386 | ........... | ........... | .556 |
| 7000 | $6.03\times10^{2}$ | $4.356\times10^{3}$ | .1385 | ........... | ........... | .554 |
| 8000 | $9.59\times10^{2}$ | $7.432\times10^{3}$ | .1290 | ........... | ........... | .553 |
| 10 000 | $1.84\times10^{3}$ | $1.814\times10^{4}$ | .1014 | ........... | ........... | .550 |

c Mean=0.001627=614.6 lumens=48.9 cpw.

## III. RADIANT LUMINOUS EFFICIENCY OF A BLACK BODY

The radiant luminous efficiency of an incandescent body is the ratio of the luminous flux to the total power radiated or the ratio of the luminous intensity to the total intensity of radiation, and for a black body the *luminous efficiency* =

$$\int_o^\infty V_\lambda\, E_\lambda\, d\lambda \div \int_o^\infty E_\lambda\, d\lambda = L \div \sigma_o T^4 \tag{6}$$

where $E_\lambda$ is given by Planck's equation and $L$ is given in Table 1 as computed from equation (5). It is of great importance in connection with the question of efficiency in light production. Experimental determinations of the radiant luminous efficiency of illuminants have occupied the attention of numerous investigators [8] and for a black body numerous computations [9,6] of this ratio have been made. The present computations (Table 1) are from the values given in the second column, which are based upon the value $\sigma = 5.7 \times 10^{-12}$ watt per cm² per deg.[4] for the coefficient of total radiation. The results are of interest in showing that the maximum luminous efficiency of a black body is attained slightly above 6000° C and has the numerical value about 14 per cent.

## IV. MECHANICAL EQUIVALENT OF LIGHT DETERMINED FROM BLACK-BODY RADIATION

One of the important applications of the equation of the visibility curve is in determining the factor for converting radiant energy into visual sensation or "light."

The unit of power is the watt. The present arbitrary practical unit of luminous flux is the lumen. The ratio of these two factors for light of maximum visibility is the stimulus coefficient or numerical factor $V_m$ in equation (1) for evaluating the lumen (or candle) in watts of luminous flux.

The reciprocal of the coefficient $V_m$ is commonly called the mechanical equivalent of light.[d] More properly it should be termed the "least mechanical equivalent of light," in view of the fact that it is the ratio of the candle (or lumen) to the watt for monochromatic radiation having the wave length of maximum visibility.

It is possible to determine the mechanical equivalent of light [10,11] from the radiation constants of a black body provided we know its brightness at a given temperature. Such determinations of brightness have been made by various observers,[12,13] the most recent being by Hyde[14] and his collaborators. The least mechanical equivalent, $m$, can be computed from the data just mentioned (Table 1) on the luminous intensity of a black body at a given temperature, at which the brightness $b$ (in candlepower per cm²) has been measured. The mathematical formula is

$$m=\frac{L}{b} \tag{7}$$

The mechanical equivalent, therefore, depends largely upon the accuracy with which these constants are known[7] and of course upon the accuracy of the visibility curve. The values of the least mechanical equivalent given in Table 1 are easily computed by dividing the luminosity values, $L$, by the brightness values, $b$, published by Hyde and his collaborators.[14] It is to be noticed that the value of $m$, as obtained at various temperatures, is subject to great variations.[e] Because of such great variations it is proposed to make all the measurements radiometrically,[11] in view

[d] This term is considered a misnomer. Unlike the mechanical equivalent of heat, the actual power equivalent of a lumen is not a fixed quantity but varies with the wave length of the radiation concerned and to a slight extent with the intensity. A more precise term for $V_m$ would be "the luminous equivalent of radiation of maximum visibility."

[e] These computations are based upon an unpublished revision of the brightness data,[14] kindly furnished by Dr. Hyde.

of the fact that the physical photometer [15,16] has been found to give excellent results.

The mean value of the least mechanical equivalent, using all of the candlepower observations,[e] is:

1 lumen = 0.001627 watt of luminous flux.
1 watt of radiation of maximum visibility = 614.6 lumens = 48.9 candles.

The candlepower measurements should be most accurate between 1700° and 2300° Abs., within which range there is no marked color difference between the comparison lamp and the black body, also the temperature scale is not in question. Using the brightness data for this temperature range, the mean value of the least mechanical equivalent is about 0.7 per cent higher, viz:

1 lumen = 0.001638 watt of luminous flux.
1 light watt = 611 lumens = 48.6 candles.

The published note [14] gives a value of 760 lumens per watt or 1 lumen = 0.00132 watt of luminous flux, the computations being based upon the older visibility curves. It is therefore of interest to recall that the value of the mechanical equivalent determined experimentally [17] was 1 lumen = 0.00162 watt. A subsequent investigation by Ives and Kingsbury [15] showed that the visibility curves then available gave results which were inconsistent with other experimental data. They found, experimentally, the visibility curve required in order to give consistent results, and on the basis of this experimental curve the old value of 1 lumen = 0.00162 watt was corrected to 1 lumen = 0.00159 watt, which differs by about 2 per cent from the present determination.

The new visibility curve is about 1 per cent larger than the one (corresponding to the absorbing solution) used by Ives and Kingsbury [15] in obtaining the value of 0.00159. Increasing this value by 1 per cent, which is the difference in the visibility curves, gives a value of

1 lumen = 0.001606 watt of luminous flux.

In subsequent investigations by Ives and Kingsbury[10,18] this constant has been checked in various ways, and they have concluded that the value 0.00159 is substantially correct.[f]

Probably one of the most trustworthy direct determinations yet made of the least mechanical equivalent of light is that by Ives,

[f] Some of their apparently most reliable measurements indicate a somewhat lower value, but not as low as 0.0012[4, 19] watt, which value is inconsistent with other experimental data.

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Coblentz, and Kingsbury,[17] in which the radiation from the monochromatic green mercury line $\lambda = 0.5461\mu$ was evaluated. Indirectly this measurement represents the mean of 61 observers, which number is sufficient for comparing results with the present work.

The observed value[17] was 1 watt = 613.6 lumens of green mercury radiation. The present visibility curve gives a value of 98.5 per cent for the visibility at $\lambda = 0.5461\mu$. Hence, correcting the observed value by 1.5 per cent we have—

1 light watt = 622.8 lumens = 49.6 candles.

1 lumen = 0.001606 watt of luminous flux of maximum visibility.

This is in exact agreement with the value obtained by using the visibility-curve solution with the physical photometer.[17,15] This is to be expected in view of the fact that, in the determinations of the mechanical equivalent from measurements on the green mercury line, the radiation constants adopted for use in calibrating the radiometer were the same as used in the present work. Hence, the only outstanding errors would be experimental ones (also the uncertainty of the value of the constant $c_2 = 14\ 350$), which should be small, owing to the large number of observers used in both investigations.[g]

It is important to remember that this value of the least mechanical equivalent of light is defined by the present visibility curve obtained under the conditions imposed as regards size and brightness of the illuminated field.

As mentioned elsewhere, an important application of the value of the mechanical equivalent of light of some spectral line is in the calibration of radiometers (in absolute measure) used in measuring light stimuli which are employed in various physiological and biological investigations involving photometric measurements. For this purpose the green mercury radiation ($\lambda = 0.546\mu$) may be used on the basis that 1 cp = 0.02 watt (1 cp = $4\pi$ lumens).

A common definition of "white light" is the light emitted by a black body at 6000° Abs. At this temperature the radiant luminous efficiency is 13.9 per cent. The radiant luminous efficiency of a black body as measured by the average eye is a maximum (14 per cent) at a temperature of about 6300° Abs. Hence, "white light" may be defined as the light emitted by a black body at 6000° C.

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[g] In a recent communication, Gerlach (Ann. der Phys., **50**, p. 259; 1916) gives a new determination of $\sigma = 5.85 \times 10^{-12}$ watt per cm² per deg.[4] This would indicate a value of 1 lumen = 0.0017 to 0.0018 watt. This value of $\sigma = 5.85 \times 10^{-12}$ does not harmonize with what one would expect from other data.

The mechanical equivalent of white light so defined is seven times as large as that of the light having a maximum luminous efficiency. The efficiency of such white light is only 14 per cent of that of the maximum, and may be expressed as ($49.6 \times 0.14$ or) 6.94 candles per watt.

## V. RADIANT LUMINOUS EFFICIENCY OF A TUNGSTEN LAMP AND THE MECHANICAL EQUIVALENT OF LIGHT

In connection with the value of the least mechanical equivalent of light, as derived in the preceding part of this paper, by using the radiation constants and the brightness of a black body at various temperatures, it was of interest to obtain further data upon this important constant. Accordingly, a further determination was made, using a standardized tungsten lamp (obtained from the photometric division) as a source of light.

The lamp was operated at 1.23 watts per candle (in a specified direction), giving 26.5 cp or $26.5 \times 10^{-4}$ lumens per square centimeter of surface 1 meter distant. By direct comparison with a (seasoned carbon incandescent lamp) standard of radiation [22] the intensity of the total radiation was $293.5 \times 10^{-6}$ watt per square centimeter at a distance of 1 meter from this lamp.

The radiant luminous efficiency of this lamp was determined by means of a thermopile, and the luminosity screen.[h] This screen has a transmission for different wave lengths approximately proportional to their visibilities.

A correction was made for shortening of the optical path by the cells containing the water and the solution. Corrections were made also for departure of the cell from perfect transmission at the maximum visibility and for difference in areas of the transmission curve of the solution and the visibility curve, using the spectral energy data of a tungsten lamp operated at closely the same color.[23] The total radiation from the lamp upon the thermopile, when the luminosity screen was not in place, was reduced by means of a sectored disk. The lamp was at a distance of 1.45 meters from the thermopile. A correction was made for radiation from the moving disk and from the surroundings when the lamp was not lighted. The first determination of the radiant luminous efficiency gave a value of 1.33 per cent. Some hours later, when the instruments were less disturbed by air currents, a more reliable series of measurements (68 galvanometer readings)

[h] Described in this Bulletin, 14, p. 167; 1917.

gave a value of 1.44 per cent. The weighted mean value (weight of second series = 4 times that of first series) is 1.42 per cent for the radiant luminous efficiency of a "40-watt" vacuum tungsten lamp operated at 1.23 watts per candle.

The least mechanical equivalent of light is obtained from a knowledge of the total radiation, the radiant luminous efficiency, and the candlepower of the tungsten lamp. The luminous flux per cm$^2$ on a surface at 1 meter distance was $(293.5 \times 10^{-6} \times 0.0142 =) 4.16 \times 10^{-6}$ light-watt per cm$^2$. Equating this to the value in lumens of the luminous flux ($26.5 \times 10^{-4}$ lumens $= 4.16 \times 10^{-6}$ watt) gives a value of 1 lumen = 0.00157 watt, which is in close agreement (about 3 per cent) with the value found in Table 1.

A check upon this value is obtained from a consideration of the power applied to the tungsten lamp, etc. The lamp was rated at 1.23 watts per horizontal candle. This is equivalent to 1.567 watts per mean spherical candle or 8.02 lumens per watt. This value must be increased by 7 per cent for conduction losses at the leading-in wires [24], and by 2 to 3 per cent for convection losses [25], or a total correction of about 10 per cent. This gives an efficiency of 8.82 lumens per watt. The total luminous efficiency $L_t$ is obtained by multiplying this value by the mechanical equivalent of light, or $8.82 \times 0.00162 = 0.0143$, which is nearly the weighted mean value obtained by direct experiments.

Our conclusions are therefore in agreement with Ives and Kingsbury [18] that the value of the least mechanical equivalent of light is close to 1 lumen = 0.0016 watt of radiant energy of maximum visibility.

## VI. THE CROVA WAVE LENGTH

The Crova [21] wave length is that wave length in the visible spectrum of two sources of light at which the ratio of the luminous intensities equals the ratio of their total luminous intensities. For a black body it is that wave length at which the luminous intensity varies by the same fractional part that the total luminous intensity varies for the same change in temperature. The practical application, as introduced by Crova, was to photometer the two sources by comparing their luminous intensities at this wave length and thus avoid the question of color difference which arises in heterochromatic photometry. The mathematical derivation of the formula for determining the Crova wave length ($\lambda_c$) has been given by various writers. The formula used in the present calcu-

lation of the Crova wave lengths is a modification of Kingsbury's[6] equation, using the values given in equation (2). It is

$$\lambda_0 = \frac{L}{\frac{\sigma}{\pi}} \left[ \frac{2.2877}{\left(\frac{129.05}{T}+1\right)^{205}} + \frac{0.1505}{\left(\frac{78.85}{T}+1\right)^{405}} + \frac{0.0805}{\left(\frac{23.52}{T}+1\right)^{1005}} + \frac{0.0775}{\left(\frac{13.67}{T}+1\right)^{2005}} \right]^{-1} \quad (8)$$

The results are given in the last column of Table 1.

## VII. SUMMARY

This paper gives some applications of the curve of visibility of radiation for the average eye (125 observers) to radiation problems. A mathematical equation is given of the average visibility curve. Using this visibility equation and Planck's equation of the black body, calculations are made of the luminous flux emitted by a black body at various temperatures, also the luminous efficiency, the Crova wave length, and the mechanical equivalent of light.

The visibility curve of the average eye gives a value for the least mechanical equivalent of 1 lumen = 0.001627 watt of luminous flux of maximum luminous efficiency. The two determinations of the least mechanical equivalent of light made by Ives, Coblentz, and Kingsbury are corrected. The values obtained by their two methods of measurement, using 61 observers, are in exact agreement, giving 1 lumen = 0.001606 watt.

A further determination of the least mechanical equivalent of light was made by using a standardized vacuum tungsten lamp as a source of radiation. The measurements on this lamp gave a value of 1 lumen = 0.00157 watt of radiant energy of maximum visibility. These computations are based upon the most probable values of the radiation constants: $C_2 = 14\,350$ micron deg. and $\sigma = 5.7 \times 10^{-12}$ watt per cm$^2$ per deg$^4$. On this basis the most reliable data now available indicate that the value of the luminous equivalent of radiation of maximum luminous efficiency is of the order of:

1 watt = 617 lumens = 49.1 candles.

1 lumen = 0.00162 watt of luminous flux.

Among other data this paper gives the determination of the radiant luminous efficiency of a vacuum tungsten lamp, the value being 1.42 per cent when operated at 1.23 watt per candle.

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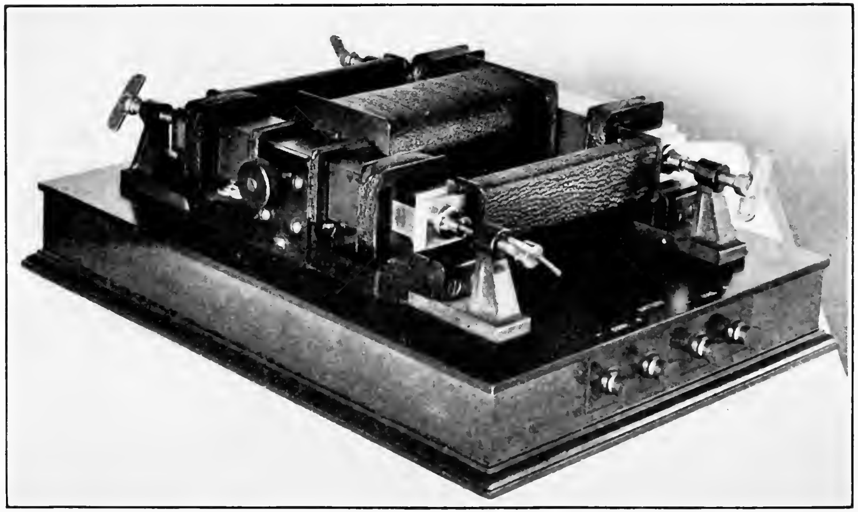

Fig. 1.—*Photograph of the Fahy permeameter*